# Kentucky Academic Standards for Mathematics: Conceptual Category Algebra

**Algebra Overview**

| Seeing Structure in Expressions | Arithmetic with Polynomials and Rational Expressions | Creating Equations ★ | Reasoning with Equations and Inequalities |
|---|---|---|---|
| • Interpret the structure of expressions.<br>• Write expressions in equivalent forms to solve problems. | • Perform arithmetic operations on polynomials.<br>• Understand the relationship between zeros and factors of polynomials.<br>• Use polynomial identities to solve problems.<br>• Rewrite rational expressions. | • Create equations that describe numbers or relationships. | • Understand solving equations as a process of reasoning and explain the reasoning.<br>• Solve equations and inequalities in one variable.<br>• Solve systems of equations.<br>• Represent and solve equations and inequalities graphically. |

**Modeling Standards**: Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice and specific modeling standards appear throughout the high school standards indicated by a star symbol (★). The star symbol sometimes appears on the heading for a group of standards; in that case, it should be understood to apply to all standards in that group.

**Plus (+) Standards:** Additional mathematics concepts students should learn in order to take advanced courses such as calculus, advanced statistics or discrete mathematics are indicated by (+) symbol.

| Algebra-Seeing Structure in Expressions | |
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| **Standards for Mathematical Practice** | |
| MP.1. Make sense of problems and persevere in solving them.<br>MP.2. Reason abstractly and quantitatively.<br>MP.3. Construct viable arguments and critique the reasoning of others.<br>MP.4. Model with mathematics. | MP.5. Use appropriate tools strategically.<br>MP.6. Attend to precision.<br>MP.7. Look for and make use of structure.<br>MP.8. Look for and express regularity in repeated reasoning. |

**Cluster: Interpret the structure of expressions**.

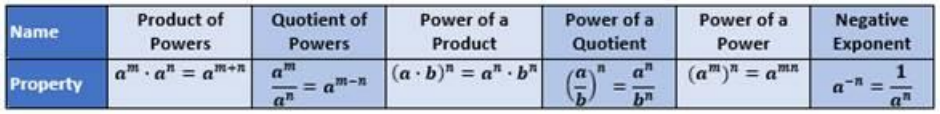
| Standards | Clarifications |
|---|---|
| KY.HS.A.1 interpret expressions that represent a quantity in terms of its context. ★<br>a. Interpret parts of an expression, such as terms, factors and coefficients.<br>b. Interpret complicated expressions, given a context, by viewing one or more of their parts as a single entity.<br>**MP.2**, **MP.6** | Students encounter simpler scenarios where they interpret r·t as the product of a given rate and time or interpret the perimeter expression (2*l*+2*w*) contextually as the sum of twice the length and twice the width of a rectangle. Students encounter more complicated scenarios where they interpret $P(1+r)^n$ contextually as the product of a principal investment, P and $(1+r)^n$ which represents an investment rate, compounding factor and time. |
| KY.HS.A.2 Use the structure of an expression to identify ways to rewrite it and consistently look for opportunities to rewrite expressions in equivalent forms.<br>**MP.7**, **MP.8** | Students see $x^4 – y^4$ as $(x^2)^2 – (y^2)^2$, thus recognizing it as a difference of squares factored as $(x^2 – y^2)(x^2 + y^2)$. Additionally, students see there are three commonly used forms for a quadratic expression:<br>• Standard form<br>• Factored form<br>• Vertex form<br>and can identify when one form might be more useful than another. |

**Attending to the Standards for Mathematical Practice**

Students not only simplify problems, they use vocabulary, such as terms, coefficients and degrees, appropriately as they describe their process ( ). Students describe the meaning of parts of an expression, such as a particular term or coefficient and also explain the meaning of the full expression ( ). Students fluently manipulate expressions into equivalent forms, based on patterns they have noticed across problems ( ).

| Algebra-Seeing Structure in Expressions | |
|---|---|
| **Standards for Mathematical Practice** | |
| MP.1. Make sense of problems and persevere in solving them.<br>MP.2. Reason abstractly and quantitatively.<br>MP.3. Construct viable arguments and critique the reasoning of others.<br>MP.4. Model with mathematics. | MP.5. Use appropriate tools strategically.<br>MP.6. Attend to precision.<br>MP.7. Look for and make use of structure.<br>MP.8. Look for and express regularity in repeated reasoning. |

**Cluster: Write expressions in equivalent forms to solve problems.**

| Standards | Clarifications |
|---|---|
| KY.HS.A.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. ★<br>a. Write the standard form of a given polynomial and identify the terms, coefficients, degree, leading coefficient and constant term.<br>b. Factor a quadratic expression to reveal the zeros of the function it defines.<br>c. Use the properties of exponents to rewrite exponential expressions.<br>d. (+) Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.<br>**MP.5**, **MP.7** | KY.HS.A.3b Students recognize the connection between the zero product property and solving a quadratic in one variable by setting factored expressions equal to zero.<br>KY.HS.A.3c<br>  <br><br>KY.HS.A.3d (+) Students recognize being able to complete the square allows them to identify the coordinates of the maximum or minimum value more easily than when the quadratic is in standard form and there are pros and cons of each equivalent form. |
| KY.HS.A.4 (+) Derive the formula for the sum of a finite geometric series (when the common ratio is not 1) and use the formula to solve problems. ★<br>**MP.1**, **MP. 4** | $S_n = \frac{a_1 - a_1 r^n}{1-r}$ where r ≠1 |

**Attending to the Standards for Mathematical Practice**

Students explain that they need to rewrite quadratic expressions into equivalent factored forms in order to find the zeros of the function it defines (  ). Using technology, students change the exponents to reinforce their understanding of exponent properties (  ).

*The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.*

| Algebra-Arithmetic with Polynomials and Rational Expressions | |
|---|---|
| **Standards for Mathematical Practice** | |
| MP.1. Make sense of problems and persevere in solving them.<br>MP.2. Reason abstractly and quantitatively.<br>MP.3. Construct viable arguments and critique the reasoning of others.<br>MP.4. Model with mathematics. | MP.5. Use appropriate tools strategically.<br>MP.6. Attend to precision.<br>MP.7. Look for and make use of structure.<br>MP.8. Look for and express regularity in repeated reasoning. |

| **Cluster: Perform arithmetic operations on polynomials.** | |
|---|---|
| **Standards** | **Clarifications** |
| KY.HS.A.5 Add, subtract and multiply polynomials.<br>**MP.7**, **MP.8** | Students combine like terms and make use of the distributive property when adding, subtracting and multiplying polynomials. |
| **Attending to the Standards for Mathematical Practice** | |
| Students flexibly rewrite expressions in equivalent forms using algebraic properties, including properties of addition, subtraction and multiplication (  ). When multiplying binomials, students identify and describe shortcuts after noticing that calculations are repeated (  ). | |

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| Algebra-Arithmetic with Polynomials and Rational Expressions | |
|---|---|
| **Standards for Mathematical Practice** | |
| MP.1. Make sense of problems and persevere in solving them.<br>MP.2. Reason abstractly and quantitatively.<br>MP.3. Construct viable arguments and critique the reasoning of others.<br>MP.4. Model with mathematics. | MP.5. Use appropriate tools strategically.<br>MP.6. Attend to precision.<br>MP.7. Look for and make use of structure.<br>MP.8. Look for and express regularity in repeated reasoning. |

**Cluster: Understand the relationship between zeros and factors of polynomials.**

| Standards | Clarifications |
|---|---|
| KY.HS.A.6 (+) Know and apply the Remainder Theorem.<br>**MP.1**, **MP.8** | Students connect long division of polynomials with the long-division algorithm of arithmetic and perform polynomial division in an abstract setting to derive the standard polynomial identities.<br><br>For a polynomial $p(x)$ and a number $a$, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$. |
| KY.HS.A.7 Identify roots of polynomials when suitable factorizations are available. Know these roots become the zeros (x-intercepts) for the corresponding polynomial function.<br>**MP.2**, **MP.5, MP.7** | Methods of finding roots could include, but are not limited to:<br>• factoring<br>• synthetic division<br>• long division<br>• an analysis of the graph (created by hand or through use of technology). |

**Attending to the Standards for Mathematical Practice**

Students reason quantitatively as they select a method for finding roots and justify why they selected and applied a particular method ( ). Students use technology to identify the *x*-intercepts from a polynomial graph and explain that the *x*-intercepts are zeros and therefore roots of the polynomials ( ).

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| Algebra-Arithmetic with Polynomials and Rational Expressions | |
|---|---|
| **Standards for Mathematical Practice** | |
| MP.1. Make sense of problems and persevere in solving them.<br>MP.2. Reason abstractly and quantitatively.<br>MP.3. Construct viable arguments and critique the reasoning of others.<br>MP.4. Model with mathematics. | MP.5. Use appropriate tools strategically.<br>MP.6. Attend to precision.<br>MP.7. Look for and make use of structure.<br>MP.8. Look for and express regularity in repeated reasoning. |
| **Cluster: Use polynomial identities to solve problems.** | |
| **Standards** | **Clarifications** |
| KY.HS.A.8 (+) Prove polynomial identities and use them to describe numerical relationships.<br>**MP.2**, **MP.3**, **MP.6** | Students observe the polynomial identity $(x^2 + y^2)^2 = (x^2 – y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples. |
| KY.HS.A.9 (+) Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of *x* and *y* for a positive integer *n*, where *x* and *y* are any numbers, with coefficients determined for example by Pascal's Triangle.<br>**MP.7**, **MP.8** | Students understand the Binomial Theorem can be proved by mathematical induction or by a combinatorial argument. |

| Algebra-Arithmetic with Polynomials and Rational Expressions | |
|---|---|
| **Standards for Mathematical Practice** | |
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**Cluster: Rewrite rational expressions.**

| Standards | Clarifications |
|---|---|
| KY.HS.A.10 (+) Rewrite simple rational expressions in different forms.<br>**MP.7**, **MP.8** | Students observe how to write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$ and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$.<br>Methods of rewriting rational expressions could include, but are not limited to:<br>• Inspection<br>• Synthetic division<br>• Long division<br>• Use of technology |
| KY.HS.A.11 (+) Add, subtract, multiply and divide rational algebraic expressions.<br>**MP.2**, **MP.3** | Students go beyond demonstrating procedural fluency and apply this standard in a variety of contextual situations. |

| Algebra-Creating Equations ★ | |
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| **Standards for Mathematical Practice** | |
| MP.1. Make sense of problems and persevere in solving them.<br>MP.2. Reason abstractly and quantitatively.<br>MP.3. Construct viable arguments and critique the reasoning of others.<br>MP.4. Model with mathematics. | MP.5. Use appropriate tools strategically.<br>MP.6. Attend to precision.<br>MP.7. Look for and make use of structure.<br>MP.8. Look for and express regularity in repeated reasoning. |
| **Cluster: Create equations that describe numbers or relationships.** | |
| **Standards** | **Clarifications** |
| KY.HS.A.12 Create equations and inequalities in one variable and use them to solve problems.<br>**MP.1**, **MP.4** | Students use the addition, subtraction, multiplication and division properties for both equations and inequalities to solve problems. These equations may arise from linear and quadratic functions and simple rational and exponential functions. |
| KY.HS.A.13 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.<br>**MP.2**, **MP.5** | Students solve systems of equations with two or more variables to solve problems in the real world setting. |
| KY.HS.A.14 Create a system of equations or inequalities to represent constraints within a modeling context. Interpret the solution(s) to the corresponding system as viable or nonviable options within the context.<br>**MP.4**, **MP.5** | Students may be asked to find an optimal solution and the conditions under which the optimal solution would occur for a given real world situation. |
| KY.HS.A.15 Rearrange formulas to solve a literal equation, highlighting a quantity of interest, using the same reasoning as in solving equations.<br>**MP.2**, **MP.7** | Students encounter scenarios where they rewrite formulas/equations for variables different from the commonly used formulas. An example may include, but not being limited to, students rearranging Ohm's law (V = IR) to highlight resistance R, rather than the variable for voltage V. |
| **Attending to the Standards for Mathematical Practice** | |
| Students interpret a story or situation into an equation or inequality, connecting the terms and symbols within the equation or inequality to the context (  ) and relate how the solution to the equation or inequality connects back to the original problem (  ). Students utilize technology to graph equations and use the graph to describe qualitatively and quantitatively the relationship between variables (  ). Students explain when they would opt for different equivalent forms an equation (  ). | |

*The identified mathematical practices, coherence connections and clarifications are possible suggestions; however, they are not the only pathways.*

| Algebra-Reasoning with Equations and Inequalities | |
|---|---|
| **Standards for Mathematical Practice** | |
| MP.1. Make sense of problems and persevere in solving them.<br>MP.2. Reason abstractly and quantitatively.<br>MP.3. Construct viable arguments and critique the reasoning of others.<br>MP.4. Model with mathematics. | MP.5. Use appropriate tools strategically.<br>MP.6. Attend to precision.<br>MP.7. Look for and make use of structure.<br>MP.8. Look for and express regularity in repeated reasoning. |
| **Cluster: Understand solving equations as a process of reasoning and explain the reasoning.** | |
| **Standards** | **Clarifications** |
| KY.HS.A.16 Understand each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.<br>**MP.1**, **MP.3** | Students reason with and about collections of equivalent expressions to see how all the expressions in the collection are linked together through the properties of operations. They discern patterns in sequences of solving equation problems that reveal structures in the equations themselves: $2x+ 4 = 10$, $2(x- 3) + 4 = 10$, $2(3x - 4) + 4 = 10$, etc.<br><br>After solving many linear equations in one variable, students look for general methods for solving a generic linear equation in one variable by replacing the numbers with letters: $ax + b= cx + d$. They have opportunities to pay close attention to calculations involving the properties of operations, properties of equality and properties of inequality as they find equivalent expressions and solve equations, noting common ways to solve different types of equations. |
| KY.HS.A.17 Solve and justify equations in one variable. Justify the solutions and give examples showing how extraneous solutions may arise.<br>a. Solve rational equations written as proportions in one variable.<br>b. Solve radical equations in one variable.<br>**MP.3**, **MP.5, MP.7** | Students analyze solution sets of equations to determine processes (for example, squaring both sides of an equation) that might lead to a solution set that differs from the original equation. |
| **Attending to the Standards for Mathematical Practice** | |
| Students use properties, such as the distributive property of multiplication over addition, to describe why two expressions are equivalent. They explain their approach to a problem, as well as critique the solutions of others, comparing the different approaches in terms of whether they are | |

accurate and efficient ( ). Students approximate solutions with technology ( ). Students use structure of an equation (rational, radical), to determine an efficient strategy for finding a solution, if one exists ( ).

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| Algebra-Reasoning with Equations and Inequalities | |
|---|---|
| **Standards for Mathematical Practice** | |
| MP.1. Make sense of problems and persevere in solving them.<br>MP.2. Reason abstractly and quantitatively.<br>MP.3. Construct viable arguments and critique the reasoning of others.<br>MP.4. Model with mathematics. | MP.5. Use appropriate tools strategically.<br>MP.6. Attend to precision.<br>MP.7. Look for and make use of structure.<br>MP.8. Look for and express regularity in repeated reasoning. |

**Cluster: Solve equations and inequalities in one variable.**

| Standards | Clarifications |
|---|---|
| KY.HS.A.18 Solve linear equations and inequalities in one variable, including literal equations with coefficients represented by letters.<br>**MP.2**, **MP.7** | Students use all properties of both equations and inequalities to solve for one variable. |
| KY.HS.A.19 Solve quadratic equations in one variable.<br>a. Solve quadratic equations by taking square roots, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers $a$ and $b$.<br>b. (+) Use the method of completing the square to transform any quadratic equation in $x$ into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.<br>c. (+) Solve quadratic equations by completing the square.<br>**MP.1**, **MP.8** | Students observe that methods for solving quadratic equations are interrelated and certain situations may more appropriately call upon one method as opposed to the other methods.<br><br>b & c. (+) Students understand completing the square involves factoring and the quadratic formula is nothing more than an encapsulation of the method of completing the square. While all students are not required to be able to use completing the square as a method for solving quadratic equations, exposure to this method is needed to explain how the quadratic formula is derived. |

**Attending to the Standards for Mathematical Practice**

Students reason about which symbolic representation is needed in order to focus on a particular feature and then efficiently rewrite literal equations to feature that characteristic ( ). Students analyze the structure of a quadratic equation to determine an efficient strategy to find a solution ( ).

| Algebra-Reasoning with Equations and Inequalities | |
|---|---|
| **Standards for Mathematical Practice** | |
| MP.1. Make sense of problems and persevere in solving them.<br>MP.2. Reason abstractly and quantitatively.<br>MP.3. Construct viable arguments and critique the reasoning of others.<br>MP.4. Model with mathematics. | MP.5. Use appropriate tools strategically.<br>MP.6. Attend to precision.<br>MP.7. Look for and make use of structure.<br>MP.8. Look for and express regularity in repeated reasoning. |

**Cluster: Solve systems of equations.**

| Standards | Clarifications |
|---|---|
| KY.HS.A.20 Solve systems of linear equations in two variables.<br>a. Understand a system of two equations in two variables has the same solution as a new system formed by replacing one of the original equations with an equivalent equation.<br>b. Solve systems of linear equations with graphs, substitution and elimination, focusing on pairs of linear equations in two variables.<br>**MP.3**, **MP.6** | a. This part of the standard is not focused on the actual process of solving a system of equations, but rather the proof of the method (specifically the elimination method).<br>b. Students utilize a variety of methods to solve system of equations including graphing the system, solving using the substitution method, solving the system with elimination both with and without involving multiplication. Students recognize the conclusion of these processes may result in obtaining one solution (expressed as an ordered pair), no solution or infinitely many solutions. |
| KY.HS.A.21 Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.<br>**MP.3**, **MP.6** | Students utilize algebra techniques and graphical representations to determine points of intersection between lines and parabolas that indicate solution sets for a system of linear and quadratic equations. |
| KY.HS.A.22 (+) Use matrices to solve a system of equations.<br>a. Represent a system of linear equations as a single matrix equation in a vector variable.<br>b. Find the inverse of a matrix if it exists.<br>c. Use matrices to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater).<br>**MP.4**, **MP.7** | a. Students do not focus on the solving of the system, but rather translating between the two different representations for this part of the standard.<br>b. Methods of solving systems with matrices could include, but are not limited to:<br>• utilizing inverse matrices<br>• row reduction<br>• Cramer's rule |

**Attending to the Standards for Mathematical Practice**

Students use a variety of methods to solve systems of equations, understanding that tables and graphs may produce estimates rather than exact solutions ( ). Students construct a viable argument to justify their solution(s) in a system of equations. ( )

| Algebra- Reasoning with Equations and Inequalities | |
|---|---|
| **Standards for Mathematical Practice** | |
| MP.1. Make sense of problems and persevere in solving them.<br>MP.2. Reason abstractly and quantitatively.<br>MP.3. Construct viable arguments and critique the reasoning of others.<br>MP.4. Model with mathematics. | MP.5. Use appropriate tools strategically.<br>MP.6. Attend to precision.<br>MP.7. Look for and make use of structure.<br>MP.8. Look for and express regularity in repeated reasoning. |
| **Cluster: Represent and solve equations and inequalities graphically.** | |
| **Standards** | **Clarifications** |
| KY.HS.A.23 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane.<br>**MP.1**, **MP.4** | Students make connections between algebra and geometry within this standard. Students acquire the basic understanding that the coordinates of the points of intersection of the graphs are the pairs of values of the variables that solve the system. |
| KY.HS.A.24 Justify that the solutions of the equations $f(x) = g(x)$ are the x-coordinates of the points where the graphs of $y = f(x)$ and $y = g(x)$ intersect. Find the approximate solutions graphically, using technology or tables. ★<br>**MP.3**, **MP.5** | Students justify solutions for equations which Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential and logarithmic functions. ★ |
| KY.HS.A.25 Graph linear inequalities in two variables.<br>a. Graph the solutions to a linear inequality as a half-plane (excluding the boundary in the case of a strict inequality).<br>b. Graph the solution set to a system of linear inequalities as the intersection of the corresponding half-planes.<br>**MP.5**, **MP.6** | Students recall skills regarding graphing the solutions of a linear inequality in the coordinate plane in order to graph the solution set for a system of linear inequalities. Students utilize these skills in other standards via linear programming. |
| **Attending to the Standards for Mathematical Practice** | |
| Students explain that the solutions of a system of equations or inequalities are all the points represented on the graph and therefore, where two functions overlap illustrates solutions to two functions ( , ). Students use technology to determine solutions to a system of linear inequalities (e.g., using DESMOS or graphing calculators) ( ). | |

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